the discrepancy using the staircase approximation increases rapidly as the cell size is increased.

VI. Conclusion

A simple and effective modification to the well known locally distorted CPFDTD algorithm has been described and a step-by-step procedure for the simple generation of the modified grid has been presented. The fact that the modified scheme is formally equivalent to a passive electrical circuit consisting of capacitors and gyrators ensures that it does not suffer from the instability inherent in the nearest-neighbor approximation. The robustness, stability and accuracy of the scheme has been verified for cylindrical resonators of complex cross-section, including right-angled corners. Trials have also been performed with many other structures such as parallel-plate-waveguide containing S bends. In all cases the algorithm remained stable and accurate. It is anticipated that the added robustness which the modification provides will facilitate more widespread use of the CPFDTD algorithm. Furthermore, building on the techniques described here, it is possible to develop a stabilised CPFDTD scheme applicable to three-dimensional (3-D) problems. This extension is the subject of ongoing work and will be described in a future contribution.

Acknowledgment

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CAD Model for Coplanar Waveguide Synthesis

Tianquan Deng

Abstract—**Accurate closed-form synthesis formulas for coplanar waveguides are presented for CAD applications, which are approximated in terms of ordinary functions. These formulas are derived, based on function approximation and curve-fitting correction of quasi-static numerical results. Comprehensive comparisons have been made by using results from the quasi-static analysis, the rigorous full-wave analysis, and the experiment available in the literature. Accuracy is found to be better than 1.5 percent for the practical range. The application range of frequency is within the limits well known for quasi-static TEM approximation and can be applied up to 20 GHz.**

I. Introduction

Recent progress in monolithic microwave integrated circuits (MMIC) and millimeter-wave integrated circuits (MMWIC) has initiated an extensive study of coplanar waveguides (CPW) due to several advantages offered over conventional microstrip lines [1], [2]. These advantages include ease of parallel and series insertion of both active and passive components and high circuit density. Another feature of coplanar waveguide is that its traces can be changed to match component lead widths while keeping the characteristic impedance constant. Most of the study efforts have been directed towards the obtaining of design parameters by either full-wave numerical methods [3]–[6] or quasi-static conformal mapping methods [7]–[12]. Full-wave analyses provide high precision in a wide frequency band. On the other hand quasi-static methods lead to closed-form expressions suitable for CAD software packages and they provide a simulation accuracy comparable with full-wave methods for frequencies up to 20 GHz [1], [13]. More recently CAD models for coplanar waveguides have received considerable attention [13]–[15]. It is noted that so far, all of the conventional CAD models for coplanar waveguides are analysis models that are used to obtain electrical parameters by the use of geometrical parameters of CPW structures. No closed-form synthesis formulas for coplanar waveguides are available; in contrast, both analysis and synthesis closed-form formulas for microstrip lines have existed for a long time [16], [17]. Such closed-form synthesis formulas provide a convenient way for designers to directly obtain the physical dimensions of CPW structures for the required design specifications rather than through an iteration approach using the conventional design equations. They

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may also provide a potential approach for models of CPW taper, stub and transition and other CPW components.

In this paper, accurate closed-form synthesis formulas in terms of ordinary functions will be presented for CPW designs, which are based on function approximation and curve-fitting correction of the respective quasi-static analysis results. In order to assess the validity of the presented formulas, the synthesis results by the presented formulas will be compared with the quasi-static analysis results and the rigorous spectral-domain hybrid-mode analysis results as well as the experimental results available in the literature.

II. Synthesis Formulas

It is well known that the closed-form quasi-static analysis formula for CPW is available [7], [18]–[20]. Its synthesis form will be derived in the following.

A. Synthesis Formula (1)

After some trivial function approximations and curve-fitting corrections of quasi-static numerical results, the synthesis formulas take the following closed-form expressions:

$$\text{when}\quad \frac{S}{H} \le \frac{10}{3(1+\ln\epsilon_r)} \quad\text{and}\quad \frac{W}{H} \le \frac{80}{3(1+\ln\epsilon_r)},$$

$$\text{then}\quad W = S\cdot G(\epsilon_r,\, H,\, Z_0\, S) \tag{1}$$

with

$$G = \begin{cases} \left[\frac{1}{4}\exp\left(\frac{\pi}{4\sqrt{\epsilon_{re}}}\frac{\eta_0}{z_0}\right) + \exp\left(-\frac{\pi}{4\sqrt{\epsilon_{re}}}\frac{\eta_0}{Z_0}\right) - 1\right], \\ \quad \text{for } Z_0 < \frac{\eta_0}{\sqrt{2(\epsilon_r+1)}} \\ \left[\frac{1}{8}\exp\left(2\pi\sqrt{\epsilon_{re}}\frac{Z_0}{\eta_0}\right) - \frac{1}{2}\right]^{-1}, \\ \quad \text{for } Z_0 \ge \frac{\eta_0}{\sqrt{2(\epsilon_r+1)}} \end{cases}$$

where ϵ_{re} is relative effective dielectric constant

$$\epsilon_{re} = \epsilon_{re}(\epsilon_r, H, Z_0, S) = A\cdot B \tag{2}$$

with $A = 1 + \sqrt{2}(\epsilon_r - 1)\sqrt{\epsilon_r + 1}\frac{Z_0}{\eta_0}\frac{K(k)}{K(k')}$

$$B = \operatorname{sech}\left\{\frac{\epsilon_r^5}{4\pi(\epsilon_r+1)^6}\left(\frac{\eta_0}{Z_0}\right)^2 \exp\left[\left(1 + 0.0016\epsilon_r Z_0\frac{S}{H}\right)\cdot\ln\left(0.6+\frac{S}{H}\right)\right]\right\}$$

$$k = \frac{\exp\left(\frac{\pi(1+g)S}{2H}\right) - \exp\left(\frac{\pi S}{2H}\right)}{\max\left(\frac{\pi(2+g)S}{2H}\right) - 1}, \quad k' = \sqrt{1-k^2},$$

$$g = G|_{\epsilon_{re}=\frac{\epsilon_r+1}{2}}, \quad\text{and}\quad \eta_0 = 120\,\pi\Omega.$$

Here $K(k)/K(k')$ is the ratio of complete elliptic integrals of the first kind, and has been approximated by Hilberg [21].

Therefore, in CPW design, for a given substrate (ϵ_r, H), and a required characteristic impedance Z_0, the central conductor width W can be evaluated immediately by choosing an appropriate gap width S (shown in Fig. 1). Moreover, relative effective dielectric constant ϵ_{re} is expressed in terms of ϵ_r, H, Z_0, S rather than in terms of conventional forms of ϵ_r, H, S, W.

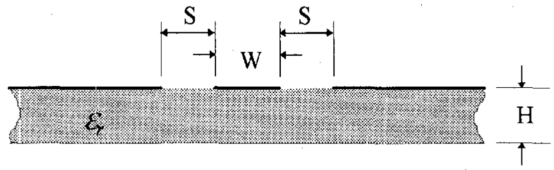

Fig. 1. The coplanar waveguide (CPW) configuration.

B. Synthesis Formula (2)

Similarly, the gap width S is determined by dielectric substrate parameters (ϵ_r, H), characteristic impedance Z_0, and the chosen central conductor width W. ϵ_{re} is expressed in terms of ϵ_r, H, Z_0, W

$$\text{when}\quad \frac{W}{H} \le \frac{80}{3(1+\ln\epsilon_r)} \quad\text{and}\quad \frac{S}{H} \le \frac{10}{3(1-\ln\epsilon_r)},$$

$$\text{then}\quad S = W/G(\epsilon_r,\, H,\, Z_0,\, W) \tag{3}$$

$$\text{and}\quad \epsilon_{re} = \epsilon_{re}(\epsilon_r, H, Z_0, W) = A\cdot B \tag{4}$$

$$\text{with}\quad B = 1 + \tanh\left\{\frac{\epsilon_r^7}{4\pi^2(\epsilon_r+1)^8}\left(\frac{\eta_0}{Z_0}\right)^2 \cdot\exp\left[\left(1+0.0004\epsilon_r Z_0\frac{W}{gH}\right)\ln\left(\frac{W}{gH}\right)\right]\right\}$$

$$k = \frac{\exp\left(\frac{\pi(1+1/g)W}{2H}\right) - \exp\left(\frac{\pi W}{2gH}\right)}{\exp\left(\frac{\pi(1+2/g)W}{2H}\right) - 1}.$$

The other variables have been given in the formula (1).

III. Numerical Results and Discussion

In order to assess the accuracy of the above synthesis formulas, comprehensive comparisons will be made by using results from the quasi-static analysis and the rigorous spectral-domain full-wave analysis as well as the experiment available in the literature.

A. Comparison with Quasi-Static Analysis

The synthesis formulas (1) and (2) are basically derived from the quasi-static analysis equations. Now the synthesis results must be compared to the analyzed ones in order to check the validity of the obtained synthesis formulas. Figs. 2 and 3, respectively, present comparisons between the analysis contours of $Z_0(S, W)$ and synthesis results $W(Z_0, S)$ by formula (1) and $S(Z_0, W)$ by formula (2) for a given dielectric substrate (ϵ_r, H). The comprehensive comparisons by using a large scale of ϵ_r and practical ranges of H, S, W and Z_0 have also been made. A similar agreement between the analysis and the synthesis results has been achieved with the accuracy of better than 1.5 percent for the practical range. The maximum error occurs when the extreme range of ϵ_r, S, W, or Z_0 is used. Self-consistent agreement has also been found between formulas (1) and (2) if Figs. 2 and 3 are compared. Note that there is no limitation for the relative dielectric constant ϵ_r of substrate when the synthesis formulas are applied. Another advantage of the synthesis formulas is that they can predict the optimized operating range of S and W for a given substrate (ϵ_r, H) and a required characteristic impedance Z_0. Such a range is the middle region (shown in Figs. 2 and 3) of S and W except for their extreme ranges.

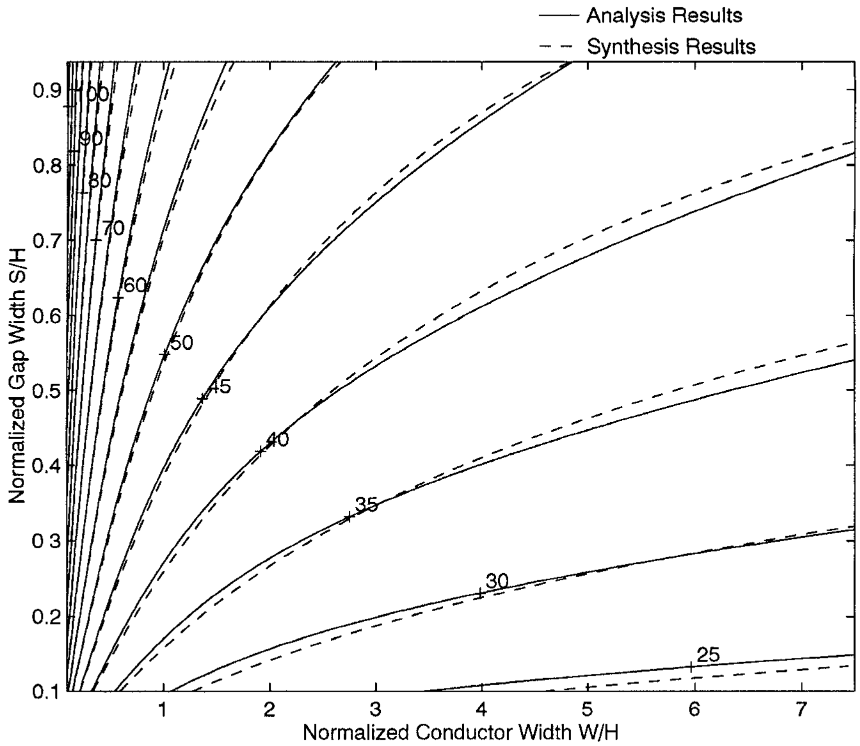

Fig. 2. Comparison between the synthesis results $W(Z_0, S)$ by formula (1) and the quasi-static analysis contours of $Z_0(S, W)$, $\epsilon_r = 12.9$, $H = 0.2$ mm.

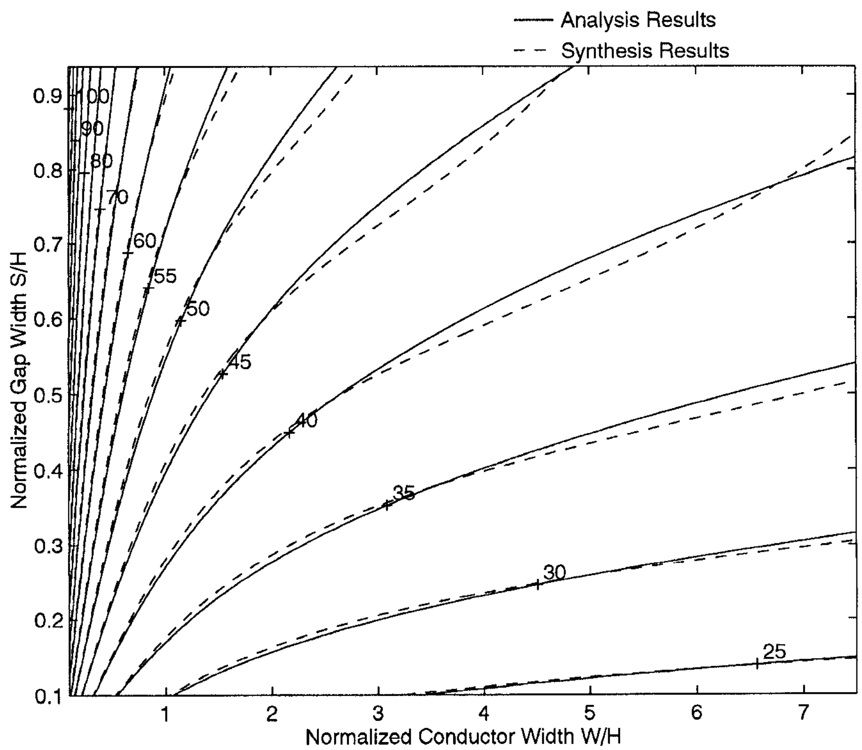

Fig. 3. Comparisons between the synthesis results $S(Z_0, W)$ by formula (2) and the quasi-static analysis contours of $Z_0(S, W)$, $\epsilon_r = 12.9$, $H = 0.2$ mm.

B. Comparison with Full-Wave Analysis

To assess the validity of the presented synthesis formulas, a comparison with full-wave results at low frequency of 1 GHz is also made. Fig. 4 show synthesis results compared with the contour of $Z_0(S, W)$ by full-wave method. This technique also provides a full-wave approach to design CPW by using the chart of $Z_0(S, W)$ contour. It is shown that a good agreement between the synthesis results and full-wave analysis results and self-consistent agreement between formulas (1) and (2) have been obtained.

Another feature of the present synthesis formulas is that relative effective dielectric constant ϵ_{re} can be directly calculated by ϵ_r, H,

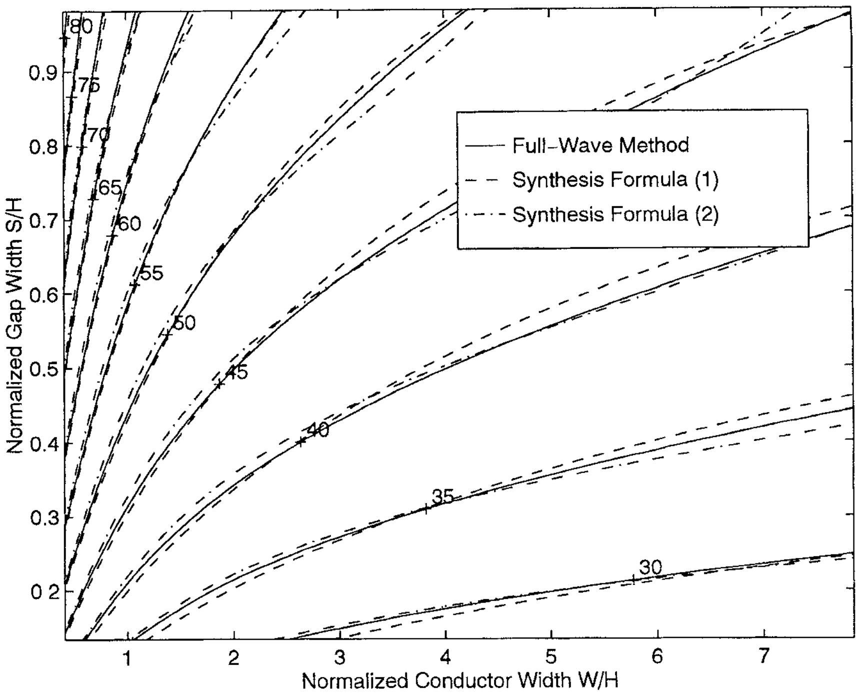

Fig. 4. Synthesis results compared with the $Z_0(S, W)$ contour of full-wave analysis results at 1 GHz, $\epsilon_r = 11$, $H = 0.381$ mm.

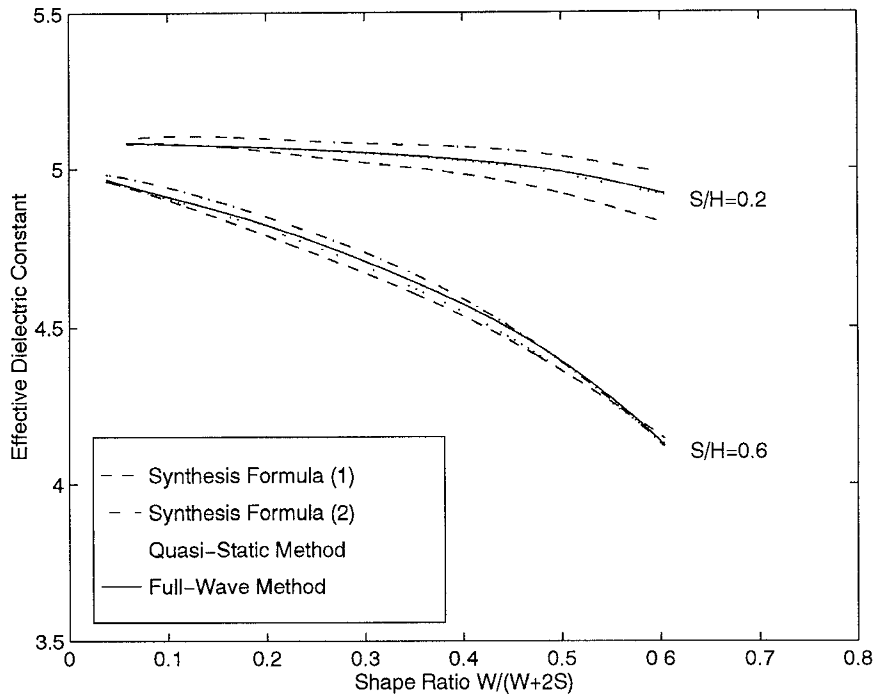

Fig. 5. Comparisons among the relative effective dielectric constant ϵ_{re} calculated by ϵ_r, H, Z_0, S; by ϵ_r, H, Z_0, W; by ε_r, H, S, W as well as by full-wave method, $\epsilon_r = 9.2$.

Z_0, S, or by ϵ_r, H, Z_0, W rather than be the conventional form of ϵ_r, H, S, W. Two cases calculated by these three equations are compared with each other and also compared with the full-wave results. The agreement is good and one example is shown in Fig. 5.

C. Comparison with other Available Results

Finally, further comparisons are presented, which include the experimental results by Dupuis *et al.* at a frequency of 2 GHz [22], and the curve-fitting results by Gupta *et al.* [23] based on Davis's numerical results [24]. A good agreement between these results is observed in Fig. 6.

D. Discussion of Application Ranges

In order to reduce dispersion and difficulty of impedance match with other lower-impedance components, the ratio of gap width S/H is chosen to be small (not more than 1.0 in common cases) in most

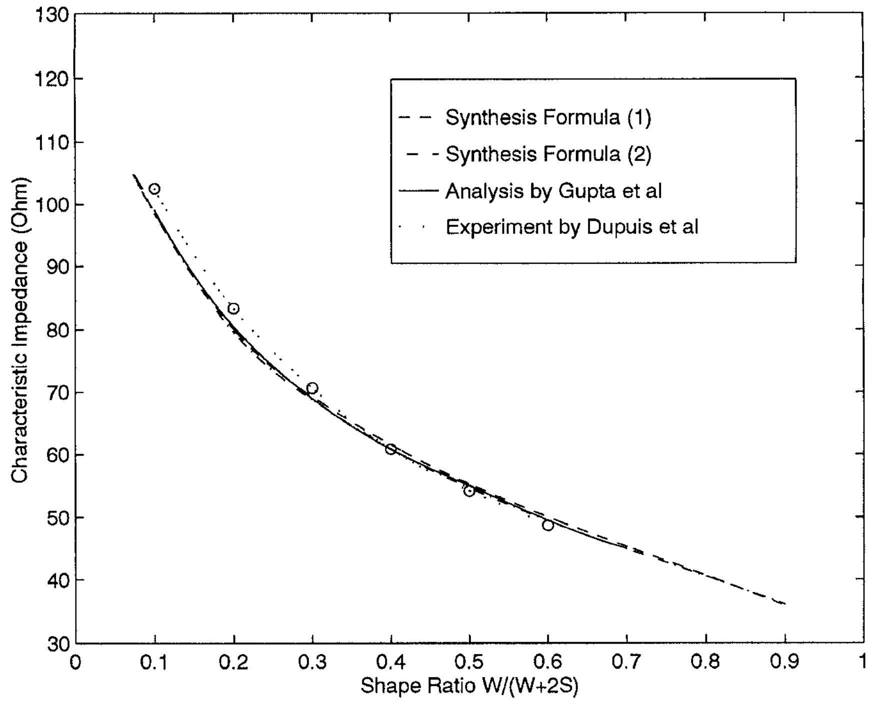

Fig. 6. Synthesis results compared with the experimental data by Dupuis *et al.* [22], and the curve-fitting analysis by Gupta *et al.* [23] based on Davis's numerical results [24], $\epsilon_r = 9.2$, $S/H = 1/3$.

of the (M)MIC design. So the application ranges provided by the synthesis formulas are broad enough for most of the CPW circuit designs especially for monolithic MIC designs. It is expected that the range is wider for lower ϵ_r, meanwhile the range is narrower for higher ϵ_r. Note that the presented formulas can be applied for a wider range of the shape ratio $W/(W+2S)$ comparable to the analysis formulas derived by Gupta *et al.* [23].

Although the presented synthesis formulas are derived with quasi-static TEM assumption and tested at low frequencies of 1 and 2 GHz, the earlier investigations [1], [13] have demonstrated that they can be used for the design of GaAs-(M)MIC's up to a frequency of 20 GHz and even up to 40 GHz. This is because the slot gap widths of CPW structures when used for MMIC's applications are typically small and the electromagnetic field is closely connected to the two slots at all frequencies and thus the dispersion of the quasi-static TEM parameters is relatively small.

IV. Conclusion

Very accurate and explicit synthesis formulas for coplanar waveguides have been presented for CAD applications, which are approximated in terms of ordinary functions. They have been tested by using the quasi-static analysis and the rigorous spectral-domain full-wave analysis as well as the available experimental results. The application range of frequency is within the limits well known for quasi-static TEM approximation and can be applied up to 20 GHz. The presented formulas provide a convenient way for designers to directly obtain the physical dimensions of CPW structures for the required design specifications rather than by the iteration approach which uses the conventional design equations. These make it easy and feasible to change the traces of CPW structures to match component lead widths while keeping the characteristic impedance constant. These may also provide a promising approach for CAD models of CPW taper, stub, and transition and other CPW components.

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Lower Order Modes of YBCO/STO/YBCO Circular Disk Resonators

Spartak Gevorgian, Erik Carlsson, Peter Linnér, Erik Kollberg, Orest Vendik, and Erland Wikborg

***Abstract*— Lower order modes in a single crystal strontium titanate (STO) circular disk resonator are studied experimentally. Superconducting epitaxial YBCO films form the parallel-plates of the resonator. Due to the extremely high dielectric constant of STO, the electric fields are concentrated between the plates, while there is a substantial magnetic fringing field which affects both the resonant frequencies, Q-factors, and tunability of all modes, especially the TM_{110} and TM_{210}.**

I. Introduction

Electrically controlled-parallel plate resonators based on bulk ferroelectric ceramics have been studied in the past [1]. Single crystal bulk strontium titanate (STO) circular disk resonators with thin, epitaxial $YBa_2Cu_3O_{7-x}$ (YBCO) plates have recently demonstrated superior performance both in the sense of higher Q-factors and controllability at frequencies between 1 and 2 GHz when cooled to liquid nitrogen (LN) temperature (77 K) [2]. These devices show a great potential for a wide range of applications in Cellular Communication and other microwave systems in the frequency range 0.5–3.0 GHz due to their low microwave losses, the electric field/temperature controllability of the STO dielectric constant around LN temperatures, and the integration of high temperature superconductors (HTS, e.g., YBCO). A drastic size reduction for microwave components in this frequency range can be achieved due to the extremely high dielectric constant of STO. Although the results reported in [2] are encouraging, an extra study of modal performance of YBCO/STO/YBCO parallel-plate resonators is needed to meet the requirements of Cellular Communication Systems and other low frequency microwave systems where size reduction and electrical tunability are critical issues. In this work we report on the results of an experimental study of the lower order modes in a YBCO/STO/YBCO parallel-plate circular disk resonator. In contrast with the work reported in [2] two lower order modes have been excited with lower resonant frequencies for the same resonator diameter offering additional size reduction and more potential in system level applications. The electric field is highly concentrated between the plates of the resonator due to the large dielectric constant while there is a substantial magnetic stray field, especially for lower order modes. This magnetic stray field strongly affects both the resonance frequency, Q-factor, and tunability of the YBCO/STO/YBCO resonator.

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TABLE I
Modal Parameters of Disk Resonator

Mode number	Modes, TM_{nm0}	k_{nm}
1	TM_{010}	0
2	TM_{110}	1.8412
3	TM_{210}	3.0542
4	TM_{020}	3.8317
5	TM_{310}	4.2012
6	TM_{410}	5.3176
7	TM_{120}	5.3314
8	TM_{510}	6.4156
9	TM_{220}	6.7061

II. Simple Theoretical Background

A. TM Modes of a Thin Parallel-Plate Circular Disk Resonator

The resonator geometry is shown in Fig. 1. Assuming a magnetic wall at the edges of the disk [1]–[3] the following equation for the resonant frequency is obtained:

$$f_{nm} = \frac{c_o k_{nm}}{2\pi r_o \sqrt{\varepsilon_r}} \tag{1}$$

with c_o being the free space light velocity, r_o the radius of the disk, and ε_r the relative dielectric constant of the dielectric (e.g. STO). k_{nm} is the mth zero of the derivative of the Bessel function